SEISMIC DAMAGE ASSESSMENT USING WAVELET ANALYSIS

Ayman. A. Shama¹

ABSTRACT

This paper presents a procedure for seismic damage assessment using wavelet analysis. The Morlet wavelet was employed in this approach as a mother wavelet and the continuous wavelet transform is conducted to decompose an acceleration signal at a discrete location in the structure into its wavelet components. Energies of the wavelet coefficients are used to compare non-damaged to damaged conditions. A parametric study was conducted using the proposed approach on a simple reinforced concrete frame structure. The nonlinear behavior of the structural elements was represented by a fiber hinge model. The structure was exposed to a suite of ground motions to characterize the extent of damage at different locations in the structure. This method proved to be as good as conventional methods for seismic damage characterization.

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Introduction

Structural damage during an earthquake is usually due to excessive deformation and hysteretic behavior under repeated load reversals. These two damage measures were the subject of numerous studies on the seismic damage assessment of reinforced concrete structures. Considerable research have been undertaken to define global damage indices for various limit states of the entire structure or locally for a structural element. Local damage in particular was the focus of many studies that were aimed towards developing local damage indices due to seismic loading in a member or at a joint. Two of the earliest forms of local damage index are stiffness degradation (Lybas and Sozen [1]) and the ductility ratio defined in terms of curvature, rotation, or displacement (Banon et al [2]; Park [3]; Sordo et al, [4]). Park and Ang [5] realized the fact that the definition of damage solely in terms of the ductility factor may be inadequate and defined a seismic damage model as a linear combination of the maximum deformation and the absorbed hysteretic energy. In the late eighties and early nineties a great deal of research was devoted towards calibrating analytical models to the hysteretic behavior of reinforced concrete obtained from laboratory experiments. Moreover, non-linear analysis programs capable of simulating this behavior were established such as DRAIN-2DX (Allahabadi and Powell [6]) and IDARC (Park et al, [7], Kunnath et al, [8]). IDARC in particular has adopted the Park-Ang damage index with slight modifications in the latest version. A number of damage index models that addressed issues such as low cycle fatigue have also been developed (Bracci et al, [9]; Chang and Mander, [10]). A common problem of these analytical damage models is that they include coefficients which must be determined either experimentally or experientially on a case by case basis. Another limitation is that these damage indices are developed to analytically predict seismic damage but not to detect it. Understanding the mechanics of seismic structural

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damage is not only significant for an ideal seismic design and effective retrofit strategies, but also essential to detect structural damage immediately following an earthquake.

It is well understood that seismic damage may alter the dynamic characteristics of the structure depending on the intensity of the ground motion and the extent of damage at potential plastic hinge zones. The extent of damage locally, which is function of the energy dissipated during the seismic event and stiffness degradation, will contribute to the global shift in the natural period of the structure. Therefore, damage can be quantified by comparing the pre-damage to post-damage energy contents of the acceleration records at a certain location within the structure. This approach may be used directly as part of a structural health monitoring program for seismic damage assessment. Wavelet analyses of accelerations at potential plastic hinge zones have been employed in this paper to evaluate the energy content and identify damage. The approach is further explained and verified through a parametric study that was conducted on a simple reinforced concrete frame structure.

**Methodology for Seismic Damage Assessment**

**Wavelet Analysis**

A wavelet is a waveform with finite energy and limited duration that has an average value of zero and has its energy localized in both frequency and space. A wavelets still has the oscillating wavelike characteristics as its Fourier transform is concentrated around a specific frequency. Wavelet analysis is carried out by breaking a signal into shifted and scaled versions of the original (mother) wavelet through continuous or discrete wavelet transform. Wavelet analysis has been employed during the past two decades in a wide range of civil engineering applications (Newland [11], Gurley and Kareem [12], Bayissa et al, [13], Shama [14]). Continuous wavelet transform is more preferable than discrete wavelet transform as it uses a wide range of frequencies in the analysis and, hence is used in the present study.

The continuous wavelet transform of a signal (CWT) is the sum over all time of the product of the signal, \( f(t) \), with translated and dilated versions of an analyzing wavelet \( \psi(t) \) as:

\[
Wf(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi^*(\frac{t-b}{a}) \, dt
\]  

(1)

Where the index \( Wf(a,b) \) comprises the wavelet coefficients, * denotes complex conjugate, a and b are scale and position parameters.

The Morlet wavelet has been used successfully in early studies on seismic data analysis. It is defined in the time domain as:

\[
\psi_0(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}
\]  

(2)

where \( \omega_0 \) and t are dimensionless frequency and time parameters. This wavelet with \( \omega_0 = 6 \)
provides a good balance between time and frequency localization and also is admissible i.e., have zero mean. Also, at this value, the Morlet wavelet scale \( a \) is almost equal to the Fourier period. The Morlet wavelet is employed in the present study as the mother wavelet for CWT.

**Wavelet Energy**

The wavelet power is used as indicator of an acceleration time history energy content. The wavelet power on the time-scale domain is defined as:

\[
p(t, f) = |W_n(ab)|^2
\]  

(3)

where \( |W_n(ab)| \) is the absolute value of the wavelet coefficient of the acceleration signal at the \( a \)th scale and \( b \)th time step. By changing the wavelet scale \( a \) and translating along the time index \( n \), a plot can be constructed to show both the wavelet power versus the scale and the variability of power with time. An example of such plot for the Northridge earthquake ground motion station USC number 0055, comp N00E, is depicted in Figure 1-b.

![Wavelet Energy](image)

Figure 1. Time frequency distribution of the wavelet power for Northridge ground motion: (a) ground motion record; and (b) wavelet power surface.

The total energy of the signal is evaluated by calculating the area under the wavelet power surface

\[
E = \sum_{m=1}^{M} \sum_{n=0}^{N-1} |W_n(ab)|^2 \, dt \, df
\]  

(4)

where \( N \) and \( M \) are the number of data points and frequencies (scales) respectively. The average energy is described as:

\[
E_{av} = \frac{E}{N}
\]  

(5)

Both \( E \) and \( E_{av} \) can be used as indices for damage identification. \( E_{av} \) in particular is viewed as the variance of the wavelet spectrum and hence is analogous to the *rms acceleration*, a universal parameter that is used to define the energy content of an earthquake ground motion.
Damage Identification

The seismic damage extent in a structural member depends on many parameters such as the peak ground acceleration, energy content, frequency content, and duration of the ground excitation; and the stiffness of the structural member. The damage index is formulated in the present study on the basis of the energy ratios obtained at the location of interest from the undamaged and damaged states of the structure. It is basically defined as the ratio of the normalized energy of the damage state to the undamaged state. Therefore the damage index at location \( j \) of the structure is expressed as:

\[
DI_j = \frac{E_{j,n}^d}{E_{j,n}^u}
\]  

(6)

where \( E_{j,n}^d \) and \( E_{j,n}^u \) are the normalized energies of the undamaged and damaged states defined as:

\[
E_{j,n}^d = \frac{E_j^d}{\sum_{j=1}^m E_j^d} \quad \text{and} \quad E_{j,n}^u = \frac{E_j^u}{\sum_{j=1}^m E_j^u}
\]  

(7)

in which \( E_j^d \) and \( E_j^u \) are the wavelet energies of the damaged and undamaged state respectively at location \( j \) and \( m \) is the number of locations in the structure.

Verification of Method

A parametric study was conducted using the proposed approach on a simple exterior frame of a three story, four-bay reinforced concrete (RC) building as illustrated in Figure 2. Dead and live loads are estimated 100 lb/ft\(^2\) and 70 lb/ft\(^2\) respectively. The structure was modeled using frame elements of the commercial software SAP2000 version 16.

Figure 2. Typical elevation and cross-sections of the moment frame used for analysis
Potential plastic hinge reinforcement details are displayed in Figure 2b. The nonlinear behavior at these locations was modeled using the fiber hinge model of SAP2000. The fiber hinge model accounts for the distributed plasticity across the cross section of the frame element. Each fiber possesses a location, a tributary area, and is represented by a stress-strain relationship. The axial stresses are integrated over the section to compute the values of the axial force and moments. Deformations and rotations are used to compute the axial strains in each fiber; and similarly the axial stresses are integrated over the section to compute axial forces and moments. The cyclic behavior of reinforced concrete was represented by the Takeda model. Locations and numbering of the potential plastic hinge zones for seismic damage assessment are depicted in Figure 3-a.

![Figure 3](image)

**Figure 3.** Node numbering and plastic collapse mechanism: (a) Numbering of potential plastic hinge zones; and (b) plastic collapse mechanism

Static pushover analysis revealed a collapse mechanism in the first floor columns and beams as illustrated in Figure 3b. Therefore, it is expected that these locations may experience high levels of damage during a seismic event. This is validated through a series of time history analyses using the energy index approach proposed in this study.

**Ground Motions**

A set of ground motions from the Northridge earthquake was selected for this parametric study. PGAs and energy contents are increasing in their values to exert different damage states in the structure. Properties and information about these ground motions are displayed in Figure 4 and Table 1.

![Figure 4](image)

**Figure 4.** Northridge Ground motions used for analyses
Table 1. Northridge earthquake ground motions employed in the seismic damage assessment

<table>
<thead>
<tr>
<th>Record Name</th>
<th>Station</th>
<th>Channel</th>
<th>PGA (g)</th>
<th>RMS acc. (g)</th>
<th>Dur. (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pasadena- Ground floor-12 S B</td>
<td>24566</td>
<td>2-180 Deg.</td>
<td>0.23</td>
<td>0.030</td>
<td>59.98</td>
</tr>
<tr>
<td>Canoga Park, CA</td>
<td>USC # 0053</td>
<td>Comp S16W</td>
<td>0.39</td>
<td>0.058</td>
<td>55.54</td>
</tr>
<tr>
<td>Newhall-LA</td>
<td>24279</td>
<td>1-90 Deg.</td>
<td>0.58</td>
<td>0.069</td>
<td>59.98</td>
</tr>
<tr>
<td>Sherman Oaks ground</td>
<td>24322</td>
<td>12-0 Deg.</td>
<td>0.80</td>
<td>0.087</td>
<td>59.98</td>
</tr>
</tbody>
</table>

Time frequency distributions of these ground motions are illustrated in Figure (5). It is observed that the energy of the earthquake is concentrated within .5 Hz to 8 Hz. Since the horizontal natural frequencies of the structure fall between 1.26 Hz and 7.5 Hz, therefore only scales equivalent to the above range of frequencies were used for the damage identification approach.

![Figure 5](image_url)  
**Figure 5.** Time-frequency distributions of the Northridge ground motions used for analyses

**Analyses and Results**

Dynamic analyses of the structure were undertaken by the time history method using the Newmark method for time integration. The viscous damping inherent in the structure was represented by Rayleigh damping wherein the mass and stiffness coefficients were set to provide an average of 2.5% damping ratio for the three fundamental horizontal periods of the structure.

Since the damage indices are calculated in terms of both the damaged and undamaged states, the ground motion for the undamaged state was selected so that it possesses the same time-frequency distribution for the exciting ground motions but with reduced amplitude and energy content that
wouldn’t impose any damage to the structure. Therefore, the Pasadena ground motion was scaled to 0.025g and used as the undamaged case for all analyses. It is believed and validated by analysis that this level of PGA will only excite the structure in the linear phase of response.

The four ground motions were applied to the structure in the horizontal direction and, for each case, the damage indices at locations 1 through 54, as indicated in Figure 3-a, were calculated using equation (6). Comparisons of these damage indices for all the ground motions are depicted in Figure 6 (refer to Figure 3A for joint numbering). It is shown that most of the damage is located in the lower floor’s columns and beams, which agrees with results from the static pushover analysis. Plastic hinge zones at the columns bases (locations 1, 7, 13, 19, 25) exhibited highest levels of damage. Damage indices for these locations ranged from 2 to 2.3. It is also clear by comparing the damage indices for different ground motions that Pasadena ground motion of PGA=0.23g imposes least damage to the structure and Sherman Oaks of PGA=0.8 exerts the highest damage. Hence, both the intensity and energy content of the ground motion are playing significant roles on the extent of damage.

Figure 6. Damage Indices at different locations of the structure for Northridge ground motions

To examine the dependence of the damage index on the energy content and PGA of the ground motion, the damage indices for each of the ground motions are displayed for hinge zone 13 in Figure 7 and the cyclic moment-curvature demands due to each of the ground motions are depicted in Figure 8.

Figure 7. Damage indices at hinge zone 13 for different ground motions
It is observed that Pasadena ground motion, which retains the lowest PGA and energy content, may cause some negligible damage in the form of minor spalling of cover concrete of the section as the maximum curvature of 0.0013 rad/ft attained by the section is less than its yield curvature of (0.0027 rad/ft). A damage index of 1.2 was reported for this ground motion. It is also noticed that as both the PGA and energy content increase the extent of damage becomes more severe. A value of 1.6 was reported for Canoga Park ground motion. This ground motion may cause minimal damage as few reinforcement rebars may yield slightly. Newhall and Sherman Oaks ground motions reported damage indices of 1.92 and 1.99. These ground motions may induce significant damage to the section as illustrated through the hysteretic cyclic performance of the hinge due to each of them.

![Energy absorption Characteristics at hinge zone 13 for different ground motions](image)

Figure 8. Energy absorption Characteristics at hinge zone 13 for different ground motions

To illustrate that the damage index is relative to the energy absorption characteristics at different locations in the structure, we display the damage indices at various hinge zones due to the Sherman Oaks ground motion in Figure 9-b. These selected locations, as marked in Figure 9-a experienced different levels of damage. The cyclic moment-curvature demands are displayed at Figure 10. By comparing Figures 9-b to 10, one can observe that the damage index increases proportionally to the energy dissipated. It is also shown that a damage index of a value less or equal to 1.0 indicates no damage; while a value that ranges from 1.0 to 1.5 indicates minor damage; and significant damage is equivalent to a damage index of a value greater than 1.5. It is apparent that that the energy dissipated through the nonlinear performance of the section is function of the section properties, reinforcement, confinement, and concrete compressive strength.
Conclusions

A new approach was proposed for seismic damage assessment. The proposed method, similar to other methods found in the literature is based on the evaluation of a damage index at a certain location within the structure. The damage index is evaluated according to this study by comparing the pre-damage to post-damage energy contents of the acceleration record at a specific location. The energy content of an acceleration record was evaluated in this study.
through continuous wavelet transform approach, wherein the universal Morlet wavelet was used as a basis function for analysis. A parametric study was performed on a simple reinforced concrete frame structure. The nonlinear cyclic behavior of reinforced concrete was represented by a Takeda model and a fiber hinge model to characterize plasticity. The structure was exposed to four ground motions that vary in their PGAs and frequency contents to characterize the extent of damage at different locations. It was found that the damage index is directly proportional to the energy content and PGA of the ground motion and to the energy absorption characteristics of the structure. For the structure considered in this study, it was found that a damage index of a value greater than 1.5 is equivalent to significant damage. It was shown that this approach proved to be as good as conventional methods for seismic damage characterization. It also may be used directly as part of a structural health monitoring program for seismic damage assessment. While the conclusions drawn based on this study are specifically for the analyzed structure, the approach can be generalized to other structures. The study emphasized the significance of the earthquake energy content. Seismic codes may need to include standard values of this parameter for design ground motions used in seismic analysis of structures. Wavelet analysis may play a significant role towards that goal.

References